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A Level Further Maths

Topic 1: Core Pure I: Proof, Complex Numbers and Matrices

AQA Further Maths (7367) | Question Paper | 100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 17 questions in this paper. The total mark is **100**.

Name: ____________________ Class: ____________

Date: ____________________ Teacher: ____________

1. This question is about the manipulation of complex numbers. Let $z_1 = 3 + 2i$ and $z_2 = 1 - 4i$.

(a) Find $z_1 + z_2$. **(1)**

(b) Find z_1z_2, giving your answer in the form $a + bi$. **(2)**

(c) Find z_1/z_2, giving your answer in the form $a + bi$. **(3)**

2. The complex number $w = -2 + 2i$ is plotted on an Argand diagram.

(a) Find the modulus $|w|$. **(2)**

(b) Find the argument $\arg(w)$, giving your answer in radians in the interval $(-\pi, \pi]$. **(2)**

(c) Hence write w in modulus-argument form. **(1)**

3. Prove the following result by mathematical induction. For all positive integers n, the sum of the first n cubes satisfies the formula below.

(a) Prove by induction that $1^3 + 2^3 + 3^3 + ... + n^3 = (1/4)n^2(n + 1)^2$ for all integers n >= 1. **(6)**

4. This question concerns the matrix A = (2 1 ; 1 3) (rows: top row 2, 1; bottom row 1, 3).

(a) Find the determinant of A. **(1)**

(b) Find the inverse matrix A^{-1}. **(2)**

(c) Hence solve the simultaneous equations $2x + y = 4$ and $x + 3y = -3$. **(3)**

5. A linear transformation of the plane is represented by the 2x2 matrix M = (0 -1 ; 1 0) (top row 0, -1; bottom row 1, 0).

(a) Describe fully the geometric transformation represented by M. **(2)**

(b) Find the image of the point (3, 1) under this transformation. **(2)**

(c) Find the matrix that represents the transformation M applied twice (i.e. M^2), and describe it geometrically. **(3)**

6. The complex number z satisfies $z = 4(\cos(\pi/6) + i\sin(\pi/6))$. Use de Moivre's theorem where appropriate.

(a) Find z^3 in modulus-argument form. **(2)**

(b) Hence express z^3 in the form a + bi. **(2)**

(c) Find $|z^5|$. **(1)**

7. This question is about the cube roots of a complex number.

(a) Solve the equation $z^3 = 8$, giving the three roots in the form a + bi where appropriate. **(4)**

(b) State the geometric configuration of the three roots when plotted on an Argand diagram. **(2)**

8. The fifth roots of unity are the solutions of $z^5 = 1$.

(a) Write down the five fifth roots of unity in the form $e^{i\theta}$ (or $\cos\theta + i\sin\theta$). **(2)**

(b) Show that the sum of all five fifth roots of unity is zero. **(3)**

9. On an Argand diagram, points are represented by complex numbers z. Sketch and describe the loci defined below (no diagram required in your answer; give the geometric description and equation).

(a) Describe the locus $|z - 3i| = 2$. **(2)**

(b) Describe the locus $|z - 2| = |z + 4|$. **(3)**

(c) Describe the locus $\arg(z - 1) = \pi/4$. **(2)**

10. The matrix B = (1 2 ; 2 4) (top row 1, 2; bottom row 2, 4) represents a transformation of the plane.

(a) Show that B is singular, and state what this implies about the transformation. **(2)**

(b) Find the equation of the line onto which the whole plane is mapped by B. **(3)**

11. A transformation T of the plane is represented by the matrix C = (3 0 ; 0 3) multiplied on the left by the reflection R = (0 1 ; 1 0); that is, T = RC where C is the top row 3, 0 and bottom row 0, 3, and R has top row 0, 1 and bottom row 1, 0.

(a) Describe the transformation represented by C. **(1)**

(b) Find the single matrix representing T = RC. **(2)**

(c) State a non-zero point that is invariant under the transformation T, or explain why none exists other than the origin. **(3)**

12. Use de Moivre's theorem to derive a multiple-angle identity.

(a) By expanding $(\cos\theta + i\sin\theta)^3$ and applying de Moivre's theorem, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **(4)**

(b) Hence, using $3\theta = \pi/3$, show that $x = \cos(\pi/9)$ is a root of the cubic equation $8x^3 - 6x - 1 = 0$. **(2)**

13. Prove the following divisibility result by induction.

(a) Prove by induction that $5^n + 2(11^n)$ is divisible by 3 for all positive integers n. **(6)**

14. The complex number $z = 1 + i\sqrt{3}$.

(a) Express z in modulus-argument form. **(3)**

(b) Use de Moivre's theorem to find z^6, giving your answer as a single real or imaginary number. **(3)**

15. This question concerns invariant lines of a transformation. The matrix D = (2 0 ; 0 3) (top row 2, 0; bottom row 0, 3) represents a stretch.

(a) Describe fully the transformation represented by D. **(2)**

(b) Show that the x-axis (the line y = 0) is an invariant line under D. **(2)**

(c) State the area scale factor of the transformation D, and justify your answer. **(2)**

16. The complex number $z = 2 - 2i$.

(a) Find the modulus and argument of z, giving the argument in the interval $(-\pi, \pi]$. **(3)**

(b) Hence, using de Moivre's theorem, find z^4 in the form $a + bi$. **(3)**

17. The complex number z satisfies the equation $z^4 = -16$.

(a) Express -16 in modulus-argument form, taking the principal argument. **(1)**

(b) Hence find all four solutions of $z^4 = -16$ in modulus-argument form. **(3)**

(c) Express the root with argument $\pi/4$ in the form $a + bi$. **(2)**