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A Level Further Maths

Topic 2: Core Pure II: Calculus, Vectors, Polar and Differential Equations

AQA Further Maths (7367) | Question Paper | 100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 17 questions in this paper. The total mark is **100**.

Name: ____________________ Class: ____________

Date: ____________________ Teacher: ____________

1. This question is about hyperbolic functions. Recall that $\cosh x = (e^x + e^{-x})/2$ and $\sinh x = (e^x - e^{-x})/2$.

(a) Find the exact value of cosh(0). **(1)**

(b) Prove the identity $\cosh^2 x - \sinh^2 x = 1$. **(3)**

2. This question concerns differentiation of hyperbolic functions.

(a) Show, from the exponential definition, that d/dx (sinh x) = cosh x. **(2)**

(b) Hence find d/dx (sinh(3x)). **(2)**

3. This question is about polar coordinates. A curve has polar equation $r = 2 + 2\cos\theta$ for $0 <= \theta < 2\pi$ (a cardioid).

(a) Find the value of r when $\theta = 0$ and when $\theta = \pi$. **(2)**

(b) State the Cartesian coordinates of the point on the curve where $\theta = \pi/2$. **(2)**

4. This question concerns improper integrals.

(a) Evaluate the improper integral ∫ from 1 to ∞ of $(1/x^2)$ dx, or state that it diverges. **(3)**

(b) Determine whether ∫ from 1 to ∞ of $(1/x)$ dx converges or diverges, justifying your answer. **(2)**

5. The region R is bounded by the curve $y = x^2$, the x-axis, and the line $x = 2$. The region is rotated 2π radians about the x-axis.

(a) Write down an integral expression for the volume of the solid of revolution generated. **(2)**

(b) Hence evaluate the volume, giving your answer as an exact multiple of π. **(3)**

6. This question is about the method of differences.

(a) Show that $1/(r(r + 1)) = 1/r - 1/(r + 1)$. **(2)**

(b) Hence use the method of differences to find the sum from $r = 1$ to n of $1/(r(r + 1))$. **(3)**

(c) Deduce the sum to infinity of the series in part (b). **(1)**

7. This question is about lines and planes in three dimensions. The line L has vector equation $r = (1, 0, 2) + \lambda(2, 1, -1)$, and the plane Π has equation $2x + y - z = 5$.

(a) Write down a normal vector to the plane Π. **(1)**

(b) Find the coordinates of the point where the line L meets the plane Π. **(4)**

(c) Find the acute angle between the line L and the plane Π. Give your answer in degrees to 1 decimal place. **(4)**

8. This question is about further vectors. Points A, B and C have position vectors a = (1, 2, 0), b = (3, 1, 2) and c = (2, 4, 1).

(a) Find the vector AB. **(2)**

(b) Find AB . AC (the scalar product of AB with AC), where AC = c - a. **(3)**

(c) Hence find the angle BAC, giving your answer in degrees to 1 decimal place. **(3)**

9. This question is about first-order differential equations. Solve the equation $dy/dx = 3x^2y$ given the boundary condition y = 2 when x = 0.

(a) Separate the variables and integrate to find the general solution. **(4)**

(b) Use the boundary condition to find the particular solution. **(2)**

10. This question is about second-order differential equations with constant coefficients. Consider the equation $d^2y/dx^2 - 5\ dy/dx + 6y = 0$.

(a) Write down the auxiliary (characteristic) equation and solve it. **(3)**

(b) Hence write down the general solution of the differential equation. **(2)**

11. A particle moves in a straight line and its displacement x metres from a fixed point O at time t seconds satisfies the equation $d^2x/dt^2 = -9x$ (simple harmonic motion).

(a) Write down the angular frequency ω and the period of the motion. **(2)**

(b) The general solution is $x = A \cos(3t) + B \sin(3t)$. Initially ($t = 0$) the particle is at $x = 4$ with velocity $dx/dt = 0$. Find A and B. **(3)**

(c) State the amplitude of the motion and the maximum speed of the particle. **(2)**

12. This question is about damped oscillations. The displacement of a damped system satisfies $d^2x/dt^2 + 4\,dx/dt + 13x = 0$.

(a) Solve the auxiliary equation and state the type of damping. **(3)**

(b) Write down the general solution of the differential equation. **(2)**

(c) State the long-term behaviour of x as $t \to \infty$, with justification. **(1)**

13. This question is about the Maclaurin series.

(a) Write down the first three non-zero terms of the Maclaurin series for e^x. **(1)**

(b) Derive the Maclaurin series for sin x up to and including the term in x^5, by evaluating successive derivatives at $x = 0$. **(4)**

(c) Hence write down an approximation for sin(0.1) using terms up to x^3, giving your answer to 5 decimal places. **(2)**

14. This question concerns arc length. A curve has equation $y = (2/3)x^{3/2}$ for $0 \le x \le 3$.

(a) Show that $dy/dx = x^{1/2}$ and hence that $1 + (dy/dx)^2 = 1 + x$. **(2)**

(b) Find the arc length of the curve from $x = 0$ to $x = 3$ using $s = \int \sqrt{(1 + (dy/dx)^2)}\, dx$. **(4)**

15. A second-order differential equation has a non-zero right-hand side: $d^2y/dx^2 - 3\, dy/dx + 2y = 4$.

(a) Find the complementary function. **(3)**

(b) Find a particular integral, and hence write the general solution. **(3)**

16. This question concerns the area enclosed by a polar curve. A curve has polar equation $r = 3 \sin \theta$ for $0 \le \theta \le \pi$.

(a) Write down the formula for the area enclosed by a polar curve between $\theta = \alpha$ and $\theta = \beta$, and write the integral needed for this curve over $0 \le \theta \le \pi$. **(2)**

(b) Using the identity $\sin^2\theta = (1 - \cos 2\theta)/2$, evaluate this area exactly. **(4)**

17. This question is about the volume of revolution about the y-axis. The region bounded by $y = x^2$, the y-axis and the line $y = 4$ is rotated 2π radians about the y-axis.

(a) Write down the integral for the volume using $V = \pi \int x^2 \, dy$, expressing x^2 in terms of y. **(2)**

(b) Hence evaluate the volume exactly. **(2)**

(c) The cylinder formed by rotating the rectangle $0 <= x <= 2$, $0 <= y <= 4$ about the y-axis has volume 16π. State the volume of the region between this cylinder and the solid found in part (b). **(2)**