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A Level Further Maths

Topic 3: Mechanics (Optional)

AQA Further Maths (7367) | Mark Scheme | 100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 11 questions in this paper. The total mark is **100**.

1. This question is about dimensional analysis. Take the dimensions of mass, length and time as M, L and T respectively.

(a) Find the dimensions of force and of energy. **(3)**

✓ force = mass × acceleration, so [force] = $M L T^{-2}$ (1)
✓ energy = force × distance (1)
✓ [energy] = $M L^2 T^{-2}$ (1)

(b) The period T of a simple pendulum is believed to satisfy $T = k l^a g^b$, where l is a length, g is an acceleration and k is dimensionless. Use dimensional analysis to find a and b. **(2)**

✓ $[T] = T = (L)^a(L T^{-2})^b = L^{a+b} T^{-2b}$, so $-2b = 1$ and $a + b = 0$ (1)
✓ $b = -1/2$, $a = 1/2$ (1)

2. A particle of mass 0.40 kg moving in a straight line has its velocity changed from 5 m s^{-1} to 13 m s^{-1} in the same direction by a constant force acting for 0.20 s.

(a) Find the magnitude of the impulse of the force on the particle. **(3)**

✓ impulse = change in momentum = m(v - u) (1)
✓ = 0.40 × (13 - 5) (1)
✓ = 3.2 N s (1)

(b) Hence find the magnitude of the constant force. **(3)**

✓ impulse = F t (1)
✓ F = 3.2 / 0.20 (1)
✓ = 16 N (1)

3. A car of mass 1200 kg accelerates from rest along a straight horizontal road. After travelling 50 m its speed is 20 m s^{-1}. The total resistance to motion is constant at 400 N.

(a) Using the work-energy principle, find the work done by the driving force over the 50 m. **(4)**

✓ work-energy: work by driving force - work against resistance = change in KE (1)
✓ change in KE = ½ × 1200 × 20^2 - 0 = 240000 J (1)
✓ work against resistance = 400 × 50 = 20000 J (1)
✓ W = 240000 + 20000 = 260000 J = 2.6×10^5 J (1)

(b) Find the power developed by the car at the instant its speed is 20 m s^{-1}, given that the driving force is constant over the 50 m. **(3)**

✓ driving force F = W / distance = 260000 / 50 = 5200 N (1)
✓ power = F v = 5200 × 20 (1)
✓ = 104000 W = 104 kW (1)

4. A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 0.8 m. The other end is fixed at a point O on a smooth horizontal table. P moves in a horizontal circle on the table with constant angular speed 3 rad s^{-1}.

(a) Find the speed of P. **(2)**

✓ v = r ω = 0.8 × 3 (1)
✓ = 2.4 m s^{-1} (1)

(b) Find the tension in the string. **(3)**

✓ tension provides the centripetal force T = m r ω^2 (1)
✓ = 0.5 × 0.8 × 3^2 (1)
✓ = 3.6 N (1)

(c) The string will break when the tension exceeds 20 N. Find the greatest angular speed at which P can move in the circle. **(3)**

✓ 20 = 0.5 × 0.8 × ω^2 (1)
✓ ω^2 = 50 (1)
✓ ω = 7.07 rad s^{-1} (3 s.f.) (1)

5. A uniform lamina occupies the region bounded by the curve y = √x, the x-axis and the line x = 4. The lamina has uniform surface density.

(a) Show that the area of the lamina is 16/3. **(4)**

✓ area = $\int_0^4 \sqrt{x}\,dx$ (1)
✓ = $\left[(2/3)\, x^{3/2} \right]_0^4$ (1)
✓ = (2/3)(8) - 0 (1)
✓ = 16/3 as required (1)

(b) Find the x-coordinate of the centre of mass of the lamina. **(4)**

✓ x-bar = ($\int_0^4 x \sqrt{x}\,dx$) / (16/3) (1)
✓ $\int_0^4 x^{3/2}\,dx = \left[(2/5)\, x^{5/2} \right]_0^4$ = (2/5)(32) = 64/5 (1)
✓ x-bar = (64/5) / (16/3) = (64/5)(3/16) (1)
✓ x-bar = 12/5 = 2.4 (1)

6. A particle moves in a straight line. At time t seconds its displacement from a fixed point O is x metres. A variable resultant force acts on the particle of mass 2 kg so that its acceleration is $a = 6t - 4$ (in m s^{-2}). When $t = 0$ the particle is at O moving with velocity 5 m s^{-1}.

(a) Find an expression for the velocity v of the particle at time t. **(4)**

✓ $v = \int a\,dt = \int (6t - 4)\,dt$ (1)
✓ $= 3t^2 - 4t + C$ (1)
✓ when $t = 0$, $v = 5$ so $C = 5$ (1)
✓ $v = 3t^2 - 4t + 5$ (1)

(b) Find the displacement of the particle from O when $t = 2$. **(4)**

✓ $x = \int v\,dt = \int (3t^2 - 4t + 5)\,dt$ (1)
✓ $= t^3 - 2t^2 + 5t + D$, with $D = 0$ since $x = 0$ at $t = 0$ (1)
✓ at $t = 2$: $x = 8 - 8 + 10$ (1)
✓ $x = 10$ m (1)

7. Two smooth spheres A and B of equal radius move along the same straight line on a smooth horizontal surface. A has mass 3 kg and speed 4 m s^{-1}; B has mass 2 kg and speed 1 m s^{-1}, moving in the same direction as A. A collides directly with B. The coefficient of restitution between the spheres is 1/2.

(a) Write down the equation of conservation of linear momentum for the collision, taking the original direction of motion as positive. **(3)**

✓ momentum before = momentum after (1)
✓ $3(4) + 2(1) = 3v_A + 2v_B$ (1)
✓ $14 = 3v_A + 2v_B$ (1)

(b) Using Newton's law of restitution, find the speeds of A and B after the collision. **(4)**

✓ restitution: $v_B - v_A = e(u_A - u_B) = (1/2)(4 - 1) = 1.5$ (1)
✓ so $v_B = v_A + 1.5$; substitute: $14 = 3v_A + 2(v_A + 1.5)$ (1)
✓ $14 = 5v_A + 3$, $v_A = 2.2$ m s^{-1} (1)
✓ $v_B = 3.7$ m s^{-1} (1)

(c) Find the loss in kinetic energy due to the collision. **(3)**

✓ KE before = $½(3)(4^2) + ½(2)(1^2) = 24 + 1 = 25$ J (1)
✓ KE after = $½(3)(2.2^2) + ½(2)(3.7^2) = 7.26 + 13.69 = 20.95$ J (1)
✓ loss = $25 - 20.95 = 4.05$ J (1)

8. A particle P of mass 0.6 kg is attached to one end of a light inextensible string of length 0.5 m. The other end is fixed at a point O. P moves in a complete vertical circle. At the lowest point of the circle P has speed 5 m s^{-1}. Take $g = 9.8 \text{ m s}^{-2}$.

(a) Find the speed of P at the highest point of the circle. **(4)**

✓ conservation of energy: $\tfrac{1}{2} m v_{top}^2 + m g (2r) = \tfrac{1}{2} m v_{bottom}^2$ (1)
✓ $v_{top}^2 = 5^2 - 2 \times 9.8 \times (2 \times 0.5)$ (1)
✓ $= 25 - 19.6 = 5.4$ (1)
✓ $v_{top} = 2.32 \text{ m s}^{-1}$ (3 s.f.) (1)

(b) Find the tension in the string at the highest point. **(4)**

✓ at the top, $T + m g = m v_{top}^2 / r$ (1)
✓ $T = m v_{top}^2/r - m g = 0.6 \times 5.4 / 0.5 - 0.6 \times 9.8$ (1)
✓ $= 6.48 - 5.88$ (1)
✓ $T = 0.60$ N (1)

(c) Find the tension in the string at the lowest point. **(4)**

✓ at the bottom, $T - m g = m v_{bottom}^2 / r$ (1)
✓ $T = m v_{bottom}^2/r + m g = 0.6 \times 25 / 0.5 + 0.6 \times 9.8$ (1)
✓ $= 30 + 5.88$ (1)
✓ $T = 35.88$ N $= 35.9$ N (3 s.f.) (1)

9. A uniform solid is formed by joining a uniform solid cylinder of radius r and height r to a uniform solid hemisphere of radius r, so that the flat face of the hemisphere coincides with one circular end of the cylinder. Both parts are made of the same material. Take the volume of a hemisphere of radius r as $(2/3)\pi r^3$ and of the cylinder as πr^3.

(a) The centre of mass of a uniform solid hemisphere of radius r is a distance 3r/8 from its flat face. Taking the centre of the flat face shared by the two parts as the origin, with positive direction into the cylinder, find the distance of the centre of mass of the combined solid from this shared face. **(5)**

✓ volumes (proportional to mass): cylinder $= \pi r^3$, hemisphere $= (2/3)\pi r^3$ (1)
✓ cylinder c.o.m. at $+r/2$ from the shared face; hemisphere c.o.m. at $-3r/8$ (1)
✓ $\bar{x} = [\pi r^3(r/2) + (2/3)\pi r^3(-3r/8)] / [\pi r^3 + (2/3)\pi r^3]$ (1)
✓ numerator $= \pi r^4(1/2 - 1/4) = \pi r^4(1/4)$; denominator $= (5/3)\pi r^3$ (1)
✓ $\bar{x} = (1/4) / (5/3) \times r = 3r/20$ from the shared face, into the cylinder (1)

(b) The combined solid is placed with the curved surface of the hemisphere on a horizontal floor and released from rest in a position slightly tilted. Explain, with reference to the centre of mass, why the solid returns to rest with the cylinder uppermost. **(4)**

✓ when resting on the hemisphere the point of contact and the centre of the sphere O are vertically below/above the line of action of weight (1)
✓ the centre of mass lies a distance 3r/20 from the flat face on the cylinder side, hence above the centre O of the hemisphere (since $3r/20 > 0$ measured into the cylinder) (1)
✓ a small tilt raises the centre of mass, creating a restoring moment about the contact point (1)
✓ therefore the equilibrium is stable and the solid returns to rest with the cylinder uppermost (1)

(c) State one modelling assumption you have made about the floor in part (b). **(4)**

✓ the floor is horizontal (1)
✓ the floor is rough enough to prevent slipping (so the solid rocks rather than slides) (1)
✓ the floor is rigid / does not deform under the solid (1)
✓ contact between the hemisphere and floor is at a single point (the surfaces are smooth/regular) (1)

10. A smooth sphere of mass 0.8 kg moving on a smooth horizontal surface has velocity (6i + 2j) m s^{-1} immediately before it is struck by an impulse. Immediately after, its velocity is (1i + 5j) m s^{-1}. The unit vectors i and j are perpendicular and horizontal.

(a) Find, in vector form, the impulse applied to the sphere. **(4)**

✓ impulse = m(v - u) (1)
✓ v - u = (1 - 6)i + (5 - 2)j = -5i + 3j (1)
✓ impulse = 0.8(-5i + 3j) (1)
✓ = (-4i + 2.4j) N s (1)

(b) Find the magnitude of the impulse. **(3)**

✓ |impulse| = $\sqrt{(-4)^2 + 2.4^2}$ (1)
✓ = $\sqrt{16 + 5.76} = \sqrt{21.76}$ (1)
✓ = 4.66 N s (3 s.f.) (1)

(c) Find the kinetic energy gained by the sphere as a result of the impulse. **(3)**

✓ KE before = ½ × 0.8 × $(6^2 + 2^2)$ = 0.4 × 40 = 16 J (1)
✓ KE after = ½ × 0.8 × $(1^2 + 5^2)$ = 0.4 × 26 = 10.4 J (1)
✓ KE gained = 10.4 - 16 = -5.6 J, i.e. a loss of 5.6 J (1)

11. A car of mass 1000 kg moves along a straight horizontal road. The engine works at a constant rate of 24 kW. The resistance to motion is constant at 600 N. Take g = 9.8 m s^{-2}.

(a) Find the acceleration of the car at the instant its speed is 10 m s^{-1}. **(4)**

✓ driving force F = P / v = 24000 / 10 = 2400 N (1)
✓ Newton's second law: F - resistance = m a (1)
✓ 2400 - 600 = 1000 a (1)
✓ a = 1.8 m s^{-2} (1)

(b) Find the maximum speed of the car on the horizontal road. **(4)**

✓ at maximum speed acceleration = 0, so driving force = resistance = 600 N (1)
✓ P = F v gives 24000 = 600 v (1)
✓ v = 24000 / 600 (1)
✓ v_{max} = 40 m s^{-1} (1)

(c) The car now ascends a hill inclined at angle θ to the horizontal, where sin θ = 1/20. With the same power and the same resistance, find the maximum speed of the car up the hill. **(5)**

✓ at maximum speed up the hill, driving force = resistance + weight component (1)
✓ weight component = m g sin θ = 1000 × 9.8 × (1/20) = 490 N (1)
✓ total opposing force = 600 + 490 = 1090 N (1)
✓ P = F v gives 24000 = 1090 v (1)
✓ v_{max} = 22.0 m s^{-1} (3 s.f.) (1)