

A Level Further Maths

Topic 3: Mechanics (Optional)

AQA Further Maths (7367)

Question Paper

100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 11 questions in this paper. The total mark is **100**.

Name: _____

Class: _____

Date: _____

Teacher: _____

1. This question is about dimensional analysis. Take the dimensions of mass, length and time as M, L and T respectively.

(a) Find the dimensions of force and of energy. (3)

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(b) The period T of a simple pendulum is believed to satisfy $T = k l^a g^b$, where l is a length, g is an acceleration and k is dimensionless. Use dimensional analysis to find a and b . (2)

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2. A particle of mass 0.40 kg moving in a straight line has its velocity changed from 5 m s^{-1} to 13 m s^{-1} in the same direction by a constant force acting for 0.20 s .

(a) Find the magnitude of the impulse of the force on the particle. (3)

(b) Hence find the magnitude of the constant force. (3)

3. A car of mass 1200 kg accelerates from rest along a straight horizontal road. After travelling 50 m its speed is 20 m s^{-1} . The total resistance to motion is constant at 400 N .

(a) Using the work-energy principle, find the work done by the driving force over the 50 m . (4)

(b) Find the power developed by the car at the instant its speed is 20 m s^{-1} , given that the driving force is constant over the 50 m . (3)

4. A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 0.8 m. The other end is fixed at a point O on a smooth horizontal table. P moves in a horizontal circle on the table with constant angular speed 3 rad s^{-1} .

(a) Find the speed of P. (2)

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(b) Find the tension in the string. (3)

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(c) The string will break when the tension exceeds 20 N. Find the greatest angular speed at which P can move in the circle. (3)

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5. A uniform lamina occupies the region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$. The lamina has uniform surface density.

(a) Show that the area of the lamina is $16/3$. (4)

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(b) Find the x-coordinate of the centre of mass of the lamina. (4)

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6. A particle moves in a straight line. At time t seconds its displacement from a fixed point O is x metres. A variable resultant force acts on the particle of mass 2 kg so that its acceleration is $a = 6t - 4$ (in m s^{-2}). When $t = 0$ the particle is at O moving with velocity 5 m s^{-1} .

(a) Find an expression for the velocity v of the particle at time t . (4)

(b) Find the displacement of the particle from O when $t = 2$. (4)

7. Two smooth spheres A and B of equal radius move along the same straight line on a smooth horizontal surface. A has mass 3 kg and speed 4 m s^{-1} ; B has mass 2 kg and speed 1 m s^{-1} , moving in the same direction as A . A collides directly with B . The coefficient of restitution between the spheres is $1/2$.

(a) Write down the equation of conservation of linear momentum for the collision, taking the original direction of motion as positive. (3)

(b) Using Newton's law of restitution, find the speeds of A and B after the collision. (4)

(c) Find the loss in kinetic energy due to the collision. (3)

8. A particle P of mass 0.6 kg is attached to one end of a light inextensible string of length 0.5 m. The other end is fixed at a point O. P moves in a complete vertical circle. At the lowest point of the circle P has speed 5 m s^{-1} . Take $g = 9.8 \text{ m s}^{-2}$.

(a) Find the speed of P at the highest point of the circle. (4)

(b) Find the tension in the string at the highest point. (4)

(c) Find the tension in the string at the lowest point. (4)

9. A uniform solid is formed by joining a uniform solid cylinder of radius r and height r to a uniform solid hemisphere of radius r , so that the flat face of the hemisphere coincides with one circular end of the cylinder. Both parts are made of the same material. Take the volume of a hemisphere of radius r as $(2/3)\pi r^3$ and of the cylinder as πr^3 .

(a) The centre of mass of a uniform solid hemisphere of radius r is a distance $3r/8$ from its flat face. (5)

Taking the centre of the flat face shared by the two parts as the origin, with positive direction into the cylinder, find the distance of the centre of mass of the combined solid from this shared face.

(b) The combined solid is placed with the curved surface of the hemisphere on a horizontal floor (4)

and released from rest in a position slightly tilted. Explain, with reference to the centre of mass, why the solid returns to rest with the cylinder uppermost.

(c) State one modelling assumption you have made about the floor in part (b). (4)

10. A smooth sphere of mass 0.8 kg moving on a smooth horizontal surface has velocity $(6\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$ immediately before it is struck by an impulse. Immediately after, its velocity is $(1\mathbf{i} + 5\mathbf{j}) \text{ m s}^{-1}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular and horizontal.

(a) Find, in vector form, the impulse applied to the sphere. **(4)**

(b) Find the magnitude of the impulse. **(3)**

(c) Find the kinetic energy gained by the sphere as a result of the impulse. **(3)**

11. A car of mass 1000 kg moves along a straight horizontal road. The engine works at a constant rate of 24 kW. The resistance to motion is constant at 600 N. Take $g = 9.8 \text{ m s}^{-2}$.

(a) Find the acceleration of the car at the instant its speed is 10 m s^{-1} . **(4)**

(b) Find the maximum speed of the car on the horizontal road. **(4)**

(c) The car now ascends a hill inclined at angle θ to the horizontal, where $\sin \theta = 1/20$. With the same power and the same resistance, find the maximum speed of the car up the hill. **(5)**
