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A Level Further Maths

Topic 4: Statistics (Optional)

AQA Further Maths (7367) | Question Paper | 100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 11 questions in this paper. The total mark is **100**.

Name: _____ Class: _____

Date: _____ Teacher: _____

1. A discrete random variable X has probability distribution given by P(X = x) = k x for x = 1, 2, 3, 4, where k is a constant.

(a) Find the value of k. **(3)**

(b) Find E(X). **(3)**

2. The number of emails arriving at an office in a randomly chosen one-minute interval is modelled by a Poisson distribution with mean 3.

(a) Find the probability that exactly 2 emails arrive in a one-minute interval. Give your answer to 4 decimal places. **(2)**

(b) Find the probability that more than 4 emails arrive in a one-minute interval. **(3)**

3. A continuous random variable X has probability density function $f(x) = k x^2$ for $0 \leq x \leq 3$, and $f(x) = 0$ otherwise.

(a) Show that $k = 1/9$. **(3)**

(b) Find $P(1 \leq X \leq 2)$. **(3)**

4. A discrete random variable Y has the probability distribution shown: P(Y = 0) = 0.2, P(Y = 1) = 0.5, P(Y = 2) = 0.3.

(a) Find E(Y) and E(Y^2). **(4)**

(b) Hence find Var(Y). **(3)**

5. At a call centre, calls of type A arrive independently as a Poisson process at a mean rate of 2 per hour, and calls of type B arrive independently as a Poisson process at a mean rate of 5 per hour.

(a) State the distribution of the total number of calls (of either type) arriving in one hour. **(2)**

(b) Find the probability that exactly 6 calls of either type arrive in one hour. **(3)**

(c) State two conditions required for the number of type A calls to be well modelled by a Poisson distribution. **(3)**

6. The continuous random variable X has probability density function $f(x) = (3/8)\,x^2$ for $0 \leq x \leq 2$, and $f(x) = 0$ otherwise.

(a) Find E(X). **(4)**

(b) Find the median value of X, giving your answer to 3 significant figures. **(4)**

7. The number of flaws per metre in a long roll of fabric is modelled by a Poisson distribution. Historically the mean number of flaws is 1.5 per metre. After a change to the manufacturing process, a manager wishes to test, at the 5% significance level, whether the mean number of flaws has decreased. In a randomly chosen one-metre length, 0 flaws are observed.

(a) State suitable null and alternative hypotheses for this test, defining any parameter you use. **(3)**

(b) Carry out the test, showing your calculation of the relevant probability. **(4)**

(c) State the conclusion of the test in the context of the question. **(3)**

8. The mass, in grams, of a randomly chosen apple from a large orchard is modelled by a normal distribution with unknown mean μ and known standard deviation 8 g. A random sample of 16 apples has mean mass 152 g.

(a) Find a 95% confidence interval for μ. Give the endpoints to 1 decimal place. **(4)**

(b) Find a 99% confidence interval for μ, using $z = 2.576$. Give the endpoints to 1 decimal place. **(4)**

(c) Explain why the 99% confidence interval is wider than the 95% confidence interval. **(3)**

9. A six-sided die is rolled 120 times. The observed frequencies of each score are: 1: 14, 2: 26, 3: 18, 4: 22, 5: 16, 6: 24. A chi-squared goodness-of-fit test is to be used, at the 5% significance level, to test whether the die is fair.

(a) State the null and alternative hypotheses, and find the expected frequency for each score under the null hypothesis. **(3)**

(b) Calculate the value of the test statistic $\Sigma (O - E)^2 / E$. **(4)**

(c) Using 5 degrees of freedom and a critical value of 11.07, complete the test and state your conclusion in context. **(4)**

(d) State the distribution that the test statistic approximately follows under the null hypothesis. **(2)**

10. A survey records whether 200 people prefer tea or coffee, classified by age group. The observed frequencies are: under 30 - tea 30, coffee 50; 30 or over - tea 60, coffee 60. A chi-squared test for independence is carried out at the 5% level to test whether drink preference is independent of age group.

(a) State the null and alternative hypotheses for the test. **(3)**

(b) Show that the expected frequency for people aged under 30 who prefer tea is 36. **(4)**

(c) The value of the test statistic is found to be 2.78. The test has 1 degree of freedom. Using a critical value of 3.841, state the conclusion of the test in context. **(3)**

(d) State why a continuity correction is sometimes applied to a 2 by 2 contingency table, and give the corrected form of one term of the test statistic. **(3)**

11. The continuous random variable X has probability density function $f(x) = (1/4)(3 - x)$ for $1 \leq x \leq 3$, and $f(x) = 0$ otherwise. (You may assume f is a valid probability density function.)

(a) Find the cumulative distribution function F(x) for $1 \leq x \leq 3$. **(4)**

(b) Find E(X). **(4)**

(c) Find $E(X^2)$ and hence Var(X). **(5)**