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A Level Further Maths

Topic 5: Discrete (Optional)

AQA Further Maths (7367) | Question Paper | 100 marks

Instructions

- Answer **all** questions. You must show sufficient working - answers with no working may score no marks.
- You may use a calculator unless a question states otherwise.
- Give exact answers where required, otherwise to an appropriate degree of accuracy.
- There are 10 questions in this paper. The total mark is **100**.

Name: ____________________ Class: ____________

Date: ____________________ Teacher: ____________

1. A simple connected graph G has 6 vertices and 9 edges.

(a) State the sum of the degrees (orders) of the vertices of G, giving a reason. **(2)**

(b) Explain what is meant by saying that G is connected. **(1)**

(c) The graph G is found to contain exactly one cycle removed from being a tree. State how many edges a spanning tree of G has, and how many edges would have to be deleted to obtain such a tree. **(2)**

(d) Explain what is meant by a bipartite graph, and state the meaning of the complete bipartite graph $K_{3,3}$. **(3)**

2. A network has five vertices A, B, C, D, E. The edge weights are: AB = 6, AC = 3, BC = 5, BD = 8, CD = 7, CE = 9, DE = 4.

(a) List the edges in order of increasing weight. **(1)**

(b) Apply Kruskal's algorithm to find a minimum spanning tree, stating clearly the order in which edges are chosen or rejected. **(4)**

(c) State the edges of the minimum spanning tree and its total weight. **(2)**

(d) Explain why a minimum spanning tree of this network must contain exactly four edges. **(2)**

3. The distances, in km, between five towns P, Q, R, S, T are given in a network. The relevant edge weights are: PQ = 10, PR = 12, PS = 16, QR = 7, QS = 14, QT = 13, RS = 9, RT = 11, ST = 8.

(a) Starting from P, apply Prim's algorithm, listing the order in which the vertices are added to the tree. **(4)**

(b) State the edges used in the minimum spanning tree and its total length. **(2)**

(c) State one practical difference between the way Prim's algorithm and Kruskal's algorithm build the tree. **(2)**

(d) State one advantage of Prim's algorithm over Kruskal's when the network is given as a distance matrix. **(1)**

4. A network of one-way road sections connects vertices A, B, C, D, E. The arc weights (travel times in minutes) are: AB = 4, AC = 2, CB = 1, BD = 5, CD = 8, CE = 10, DE = 2.

(a) Use Dijkstra's algorithm to find the shortest time from A to each other vertex. Show the final (permanent) label at each vertex. **(5)**

(b) State the shortest route from A to E and its length. **(2)**

(c) Explain how, once the algorithm is complete, the shortest route to a vertex is traced back. **(2)**

(d) State one assumption about the arc weights needed for Dijkstra's algorithm to be valid. **(1)**

5. A network of paths must be inspected by a worker who starts and finishes at the same point and walks along every path. The four odd vertices of the network are A, B, C, D, with the shortest path distances between them being AB = 8, AC = 5, AD = 9, BC = 7, BD = 6, CD = 4. The total length of all the paths is 120 m.

(a) Explain why the route inspection (Chinese postman) problem requires the odd vertices to be paired up. **(2)**

(b) List the three ways of pairing the four odd vertices and find the total of the shortest distances for each pairing. **(3)**

(c) State the minimum total distance the worker must walk to inspect every path and return to the start. **(2)**

(d) Explain the difference between the route inspection problem and the travelling salesperson problem. **(2)**

6. A salesperson must visit five towns and return to the start. A lower bound for the optimal tour is found by deleting a vertex and adding to the minimum spanning tree of the rest the two shortest arcs from the deleted vertex. The minimum spanning tree of the remaining four towns has weight 40, and the two shortest arcs from the deleted town have lengths 9 and 11.

(a) Calculate this lower bound for the length of the optimal tour. **(2)**

(b) Using a nearest neighbour algorithm from the start town a tour of length 72 is found. Write down an inequality satisfied by the length L of the optimal tour. **(2)**

(c) Explain why the nearest neighbour algorithm does not always produce the optimal tour. **(2)**

(d) Explain why the travelling salesperson problem is hard to solve exactly for large networks, and state what is meant by a heuristic algorithm in this context. **(4)**

7. This question is about standard algorithms applied to the list of numbers 8, 3, 5, 9, 2, 6.

(a) Perform one pass of a bubble sort (sorting into increasing order), writing down the list after the pass. **(3)**

(b) State how many comparisons are made in one complete pass of a bubble sort on this list of six numbers. **(1)**

(c) Boxes of capacity 10 are to be filled with items of sizes 8, 3, 5, 9, 2, 6 (in this order). Apply the first-fit algorithm and state how many boxes are used. **(3)**

(d) Apply the first-fit decreasing algorithm to the same items and capacity, and state whether it uses fewer boxes. **(3)**

8. A project consists of activities A to F with the following durations (days) and immediate predecessors: A(3, none), B(4, none), C(2, A), D(5, A and B), E(6, C), F(4, D).

(a) Find the early event times by carrying out a forward pass through the network. **(3)**

(b) Find the late event times by carrying out a backward pass, and hence state the minimum project completion time. **(3)**

(c) List the critical activities and state the critical path. **(2)**

(d) Calculate the total float on activity C. **(2)**

9. A manufacturer makes x units of product P and y units of product Q. The profit, in pounds, is given by $5x + 4y$, subject to $2x + y \le 20$, $x + 3y \le 30$, and $x, y \ge 0$.

(a) Write the linear programming problem as a maximisation in standard form, introducing slack variables s and t. **(3)**

(b) Write down the initial simplex tableau for this problem. **(2)**

(c) Identify the pivot column and pivot element for the first iteration, explaining your choice. **(3)**

(d) By solving the problem graphically or otherwise, find the values of x and y that maximise the profit and state the maximum profit. **(4)**

10. A two-player zero-sum game has the pay-off matrix for player A (row player), with player B as the column player: row strategy A1 gives (3, 1) against B's columns, and row strategy A2 gives (0, 4).

(a) Determine the row maximin and the column minimax, and hence explain whether the game has a stable solution (saddle point). **(4)**

(b) Player A plays A1 with probability p and A2 with probability $1 - p$. Write expressions for A's expected pay-off against each of B's pure strategies. **(2)**

(c) Find the value of p for A's optimal mixed strategy and the value of the game. **(3)**

(d) A separate transport network has a source S and sink T. A cut separates S from T and consists of the arcs with capacities 7, 5 and 4. State the capacity of this cut and explain what the maximum flow - minimum cut theorem tells you about the maximum flow. **(4)**