

A-Level Further Maths

Full Specification - Spec Cheat Sheet · Core + all options

AQA Further Maths (7367)

Core + all options

127 spec points

Every specification point for **Full Specification**, in spec order - tick each one off as you revise. Codes match the official AQA specification. Nothing here is optional.

Compulsory A: Proof

- A1** Build on AS/A-Level Mathematics proof; construct rigorous mathematical arguments and proofs.
- A2** Use proof by induction to establish results, for example sums of series, divisibility statements and powers of matrices.

Compulsory B: Complex numbers

- B1** Solve any quadratic equation with real coefficients; solve cubic and quartic equations with real coefficients.
- B2** Add, subtract, multiply and divide complex numbers given in the form $x + iy$, where x and y are real.
- B3** Understand and use the complex conjugate; know that non-real roots of a real-coefficient polynomial occur in conjugate pairs.
- B4** Use and interpret the Argand diagram; represent complex numbers as points and as vectors.
- B5** Work with the modulus and argument of a complex number; know the definition $|z|$ and $\arg(z)$ and the relationship to the Cartesian form.
- B6** Convert between Cartesian form and modulus-argument (polar) form; write a complex number as $r(\cos t + i \sin t)$.
- B7** Multiply and divide complex numbers in modulus-argument form, including the effect on moduli and arguments.
- B8** Construct and interpret loci in the Argand diagram, such as $|z - a| = r$ and $\arg(z - a) = t$, and regions defined by inequalities.
- B9** Understand de Moivre's theorem and apply it to trigonometric identities and to roots of complex numbers.
- B10** Know and use the exponential form $e^{i t} = \cos t + i \sin t$ for a complex number.
- B11** Find the n distinct n th roots of a non-zero complex number and know they form the vertices of a regular polygon in the Argand diagram.
- B12** Use complex roots of unity to solve geometric and other problems.

Compulsory C: Matrices

- C1** Add, subtract and multiply conformable matrices; multiply a matrix by a scalar.
- C2** Understand and use zero and identity matrices.
- C3** Use matrices to represent linear transformations in 2-D; successive transformations; single transformations in 3-D (limited to reflection in a coordinate plane and rotation about a coordinate axis).
- C4** Find invariant points and invariant lines for a linear transformation.
- C5** Calculate determinants of 2×2 and 3×3 matrices and interpret them as scale factors, including the effect on orientation.

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| C6 | Understand and use singular and non-singular matrices; properties of inverse matrices. |
| C7 | Calculate and use the inverse of a non-singular 2x2 matrix and a non-singular 3x3 matrix. |
| C8 | Solve three linear simultaneous equations in three variables by use of the inverse matrix. |
| C9 | Interpret geometrically the solution and non-existence of solutions of three simultaneous linear equations. |

Compulsory D: Further algebra and functions

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| D1 | Understand and use the relationships between the roots and coefficients of polynomial equations up to quartic equations. |
| D2 | Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation. |
| D3 | Understand and use the method of differences for the summation of series, including the use of partial fractions. |
| D4 | Find the Maclaurin series of a function, including the general term where it can be found. |
| D5 | Recognise and use the Maclaurin series for e^x , $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$, including the interval of validity. |

Compulsory E: Further calculus

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| E1 | Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity. |
| E2 | Understand and evaluate the mean value of a function over a given interval. |
| E3 | Integrate using partial fractions extended to denominators with a repeated and/or a quadratic factor. |
| E4 | Differentiate inverse trigonometric functions. |
| E5 | Integrate functions of the form $(a^2 - x^2)^{-1/2}$ and $(a^2 + x^2)^{-1}$ and be able to choose trigonometric substitutions to integrate associated functions. |
| E6 | Derive formulae for and calculate volumes of revolution about both the x-axis and the y-axis. |
| E7 | Understand and use the formula for arc length, including in Cartesian, parametric and polar form. |
| E8 | Understand and use the formula for the surface area of a surface of revolution, including in Cartesian, parametric and polar form. |

Compulsory F: Further vectors

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| F1 | Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D. |
| F2 | Understand and use the vector and Cartesian forms of the equation of a plane. |
| F3 | Calculate the scalar product and use it to express the equation of a plane and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane. |
| F4 | Check whether vectors are perpendicular by using the scalar product. |
| F5 | Find the intersection of a line and a plane; calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane. |

Compulsory G: Polar coordinates

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| G1 | Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. |
| G2 | Sketch curves with r given as a function of t , including use of trigonometric functions. |
| G3 | Find the area enclosed by a polar curve using the integral one half of the integral of r squared with respect to t . |

Compulsory H: Hyperbolic functions

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| H1 | Understand the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs. |
| H2 | Differentiate and integrate hyperbolic functions. |
| H3 | Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. |
| H4 | Derive and use the logarithmic forms of the inverse hyperbolic functions. |
| H5 | Integrate functions of the form $(x^2 + a^2)^{-1/2}$ and $(x^2 - a^2)^{-1/2}$ and be able to choose substitutions to integrate associated functions. |

Compulsory I: Differential equations

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| I1 | Find and use an integrating factor to solve first order linear differential equations and recognise when it is appropriate to do so. |
| I2 | Find both general and particular solutions of differential equations and use boundary conditions to find particular solutions. |
| I3 | Use differential equations in modelling in kinematics and in other contexts. |
| I4 | Solve differential equations of the form $y'' + a y' + b y = 0$ where a and b are constants, by using the auxiliary equation. |
| I5 | Solve differential equations of the form $y'' + a y' + b y = f(x)$ where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function. |
| I6 | Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative, and the form of the solution. |
| I7 | Solve the equation for simple harmonic motion $x'' = -k^2 x$ and relate the solution to the motion. |
| I8 | Model damped oscillations using second order differential equations and interpret the solutions for light, critical and heavy damping. |
| I9 | Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations, and be able to solve them, for example for predator-prey models. |

Compulsory J: Trigonometry

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| J1 | Understand and use the definitions of the inverse trigonometric functions, their domains and ranges, and be able to sketch their graphs. |
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Compulsory K: Numerical methods

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| K1 | Use numerical methods to find approximate solutions, including the use of the mid-ordinate rule and Simpson's rule for integration. |
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Option: Mechanics - MA Dimensional analysis

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| MA1 | Find the dimensions of a quantity in terms of mass, length and time (M, L, T). |
| MA2 | Use dimensional analysis to find the units of a quantity in terms of base SI units. |
| MA3 | Use dimensional analysis to check the consistency of, predict the form of, or determine missing exponents in physical equations. |

Option: Mechanics - MB Momentum and collisions

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| MB1 | Understand and use the definition of impulse as the change in momentum, in one and two dimensions. |
| MB2 | Understand and use conservation of momentum in two dimensions, including the use of vectors. |

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| MB3 | Understand and use Newton's experimental law and the coefficient of restitution for direct impacts and for impacts with a fixed surface. |
| MB4 | Understand and use the idea of successive impacts and apply conservation of momentum and the restitution law. |
| MB5 | Understand and use oblique impact of a smooth sphere with a fixed smooth surface and oblique impact of smooth spheres, including the impulse and the change in kinetic energy. |
| MB6 | Calculate the loss of kinetic energy due to an impact. |

Option: Mechanics - MC Work, energy and power

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| MC1 | Understand and use the work done by a force, including the work-energy principle, for a variable force. |
| MC2 | Understand and use the definitions of gravitational potential energy, kinetic energy and elastic potential energy. |
| MC3 | Understand and use the principle of conservation of mechanical energy and the work-energy principle in problems involving variable forces. |
| MC4 | Understand and use the definition of power as the rate of doing work, including for a vehicle moving against a resistance. |

Option: Mechanics - MD Circular motion

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| MD1 | Understand and use angular speed and the relationships $v = r\omega$ and $a = r\omega^2 = v^2 / r$ for motion in a circle. |
| MD2 | Model motion in a horizontal circle with constant speed, including the conical pendulum and motion on a banked surface. |
| MD3 | Understand and use the equations of motion for a particle moving in a vertical circle, including conditions for completing the circle or leaving the surface. |
| MD4 | Solve problems involving motion in a vertical circle for a particle on a string, on the inside or outside of a surface, and on a rod. |

Option: Mechanics - ME Centres of mass and moments

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| ME1 | Find the centre of mass of a system of particles and of a composite body by symmetry and by moments. |
| ME2 | Find the centre of mass of a uniform lamina, including by integration, and of a uniform arc, sector and solid of revolution. |
| ME3 | Use the position of the centre of mass to solve problems involving the equilibrium of a rigid body, including suspension and the point of toppling or sliding. |

Option: Mechanics - MF Further kinematics

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| MF1 | Understand and use velocity and acceleration as derivatives with respect to time when displacement is a function of time, in one and two dimensions, using vectors. |
| MF2 | Understand and use calculus to model motion where acceleration is a function of time, of velocity or of displacement, in a straight line. |
| MF3 | Set up and solve differential equations to model rectilinear motion under a variable force. |

Option: Statistics - SA Discrete random variables and expectation

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| SA1 | Understand and use the expectation and variance of a discrete random variable, including $E(aX + b)$ and $\text{Var}(aX + b)$. |
| SA2 | Understand and use the expectation and variance of a linear function of independent random variables, and of sums and differences of independent random variables. |
| SA3 | Use the discrete uniform distribution as a model, including its mean and variance. |

Option: Statistics - SB Poisson distribution

- SB1** Understand the conditions under which a Poisson distribution is a suitable model and use the Poisson distribution to calculate probabilities.
- SB2** Understand and use the mean and variance of the Poisson distribution.
- SB3** Understand and use the sum of independent Poisson distributions as a model.

Option: Statistics - SC Continuous random variables

- SC1** Understand and use a probability density function for a continuous random variable, including verifying that a function is a valid density function.
- SC2** Find the mean, variance, median, quartiles and mode of a continuous random variable from its probability density function.
- SC3** Understand and use the cumulative distribution function and the relationship between it and the probability density function.
- SC4** Understand and use the continuous uniform (rectangular) distribution as a model, including its mean and variance.

Option: Statistics - SD Hypothesis testing and confidence intervals

- SD1** Understand and apply the distribution of the sample mean of a sample drawn from a normal distribution with known variance.
- SD2** Carry out a hypothesis test for the mean of a normal distribution with known variance.
- SD3** Construct and interpret a confidence interval for the mean of a normal distribution with known variance.
- SD4** Carry out a hypothesis test for the mean of a Poisson distribution using a suitable approximation or exact probabilities.

Option: Statistics - SE Chi-squared tests for contingency tables and goodness of fit

- SE1** Understand and use the chi-squared statistic as a measure of goodness of fit between observed and expected frequencies.
- SE2** Carry out a goodness of fit test for given discrete and continuous distributions, including the discrete uniform, binomial, Poisson and continuous uniform, calculating expected frequencies and degrees of freedom.
- SE3** Carry out a chi-squared test for independence in a contingency table, including determining degrees of freedom and pooling classes where expected frequencies are small.

Option: Discrete - DA Graphs

- DA1** Understand and use the language of graphs, including vertices, edges, degree, walks, trails, paths, cycles, connectedness, trees, simple graphs and bipartite graphs.
- DA2** Understand and use the handshaking lemma relating the sum of vertex degrees to the number of edges.
- DA3** Understand and use adjacency matrices and distance matrices to represent a graph or network.
- DA4** Understand and use the terms Eulerian and semi-Eulerian and the conditions for a graph to be Eulerian or semi-Eulerian; understand the terms Hamiltonian and planar.

Option: Discrete - DB Networks and minimum spanning trees

- DB1** Find a minimum spanning tree of a network using Kruskal's algorithm.
- DB2** Find a minimum spanning tree of a network using Prim's algorithm in graphical form and from a distance matrix.
- DB3** Find a shortest path between two vertices in a network using Dijkstra's algorithm.

Option: Discrete - DC Route inspection and the travelling salesperson problem

- DC1** Solve the route inspection (Chinese postman) problem for a network with up to four odd vertices, finding the shortest closed route that traverses every edge.
- DC2** Understand the travelling salesperson problem and the difference between the classical and practical versions.
- DC3** Use a minimum spanning tree to find a lower bound for the travelling salesperson problem.
- DC4** Use the nearest neighbour algorithm to find an upper bound for the travelling salesperson problem.

Option: Discrete - DD Algorithms and sorting

- DD1** Understand and use the idea of an algorithm and be able to trace, implement and interpret algorithms given in words or as flow charts.
- DD2** Carry out and interpret bubble sort and quick sort algorithms on a list.
- DD3** Understand and use the order of an algorithm and compare the efficiency of algorithms in terms of their order.
- DD4** Carry out bin packing using first-fit, first-fit decreasing and full-bin strategies and understand the heuristic nature of these methods.

Option: Discrete - DE Critical path analysis

- DE1** Construct and interpret an activity network using activity-on-node, including the use of dummy activities and precedence tables.
- DE2** Carry out a forward and backward pass to find early and late event times and determine the critical activities and the critical path.
- DE3** Calculate the total float of an activity and understand its meaning.
- DE4** Construct and interpret Gantt (cascade) charts and resource histograms, and carry out resource levelling.

Option: Discrete - DF Linear programming

- DF1** Formulate a problem as a linear programming problem in two variables, identifying the objective function and constraints.
- DF2** Solve a two-variable linear programming problem graphically, using the objective line (ruler) method or the vertex method.
- DF3** Find optimal integer solutions to a linear programming problem where appropriate.
- DF4** Use the simplex algorithm and simplex tableaux to solve maximisation problems subject to constraints of the form less than or equal to, including the interpretation of slack variables.

Option: Discrete - DG Game theory and network flows

- DG1** Understand and use pay-off matrices for two-person zero-sum games; identify play-safe strategies and stable solutions (saddle points).
- DG2** Reduce a pay-off matrix using dominance arguments and find an optimal mixed strategy for a game with no stable solution by converting to a linear programming problem.
- DG3** Understand and use the language of flows in a network, including sources, sinks, capacities, cuts and the value of a flow.
- DG4** Use the maximum flow minimum cut theorem and the flow augmentation method to find the maximum flow through a network.